Optimal movement for efficient pike circle

[BRE] Gaston (Castagne)

1 Introduction

How often have you asked yourself:

1. Given a moving, two column formation, can one construct the perfect circle of pikes to protect our lord or lady from a gruesome charge of enemy cavalry?
2. Can one determine the position of such a circle, so as to safeguard our liege in the quickest manner possible and in doing so maximize soup related gain?
3. Is this something worth considering in the first place?
4. Seriously, should i stop worrying about this at 3am?

Well, to answer your questions in rigorous detail:

1. Yes.
2. Yes.
3. Probably not.
4. Most definitely.

In this scripture I will ease your mind with some long-awaited answers to your life defining questions.

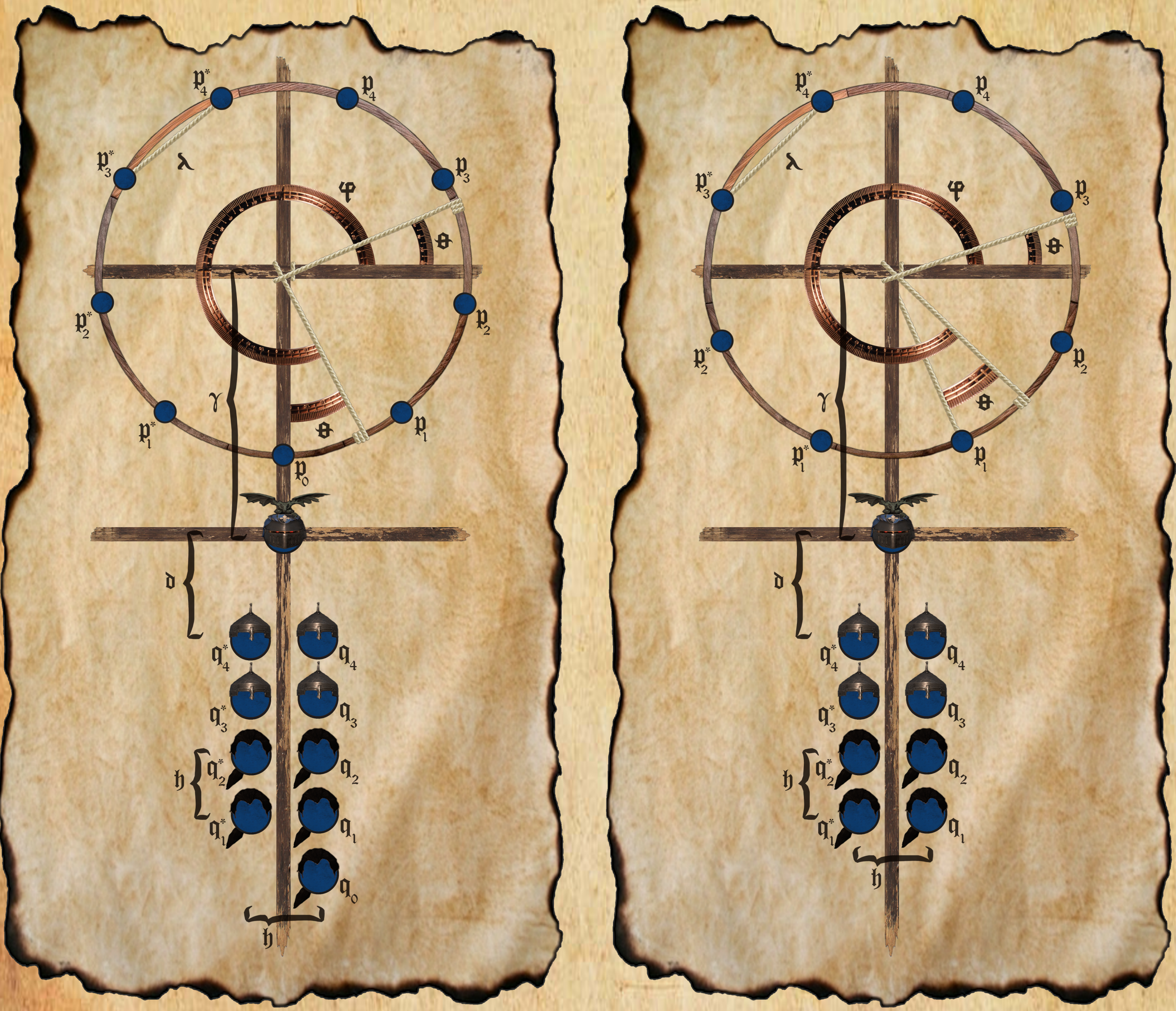
I will argue that the circle is complete when the commander is placed safely in the centre, guarded by equally distanced plebeians along the circumference.

One could possibly argue that a solution to the problem would be: The officer runs towards the first line of pikeneers, as the pikeneers construct a circle with the border closest to the officer being placed at the halfway point between their original positions. We will abandon this solution for two reasons: 1. The officer not being placed snugly in the centre invites not only harm, but also chaos (the bad kind), into our midst. 2. With this solution being utterly trivial, it would mean that someone has wasted too many hours of their life on this wall of mediocrity.

Finally: Yes, this font is tiresome to read. No, I am not sorry.

2 Setup and mathematical mumbo jumbo

In figures 1a and 1b one can see examples of the initial formation as well as the placement on the circle for odd and even number of commoners, respectively. The glorious leader is initially placed in the origin, as fits our knightocentric world view. The first row of subordinates are at a distance d behind them, and each row and column are a distance h apart. The distance between the captain's final and original position is γ. It is suggested without further proof that the placements on the circle as shown is optimal for the even case. For the odd case it is near optimal, but makes the most sense in regards to the natural instinct of the common foot soldier. It is also proposed without further proof that the optimal distribution of plebs q_i and q_j^* is for soup-chaser q_i to move to position $p_i, \forall i$, and for team-killer q_j^* to move to position $p_j^*, \forall j$. Due to the symmetry of the problem it will suffice to prove this for only one of the sides of the formation.



(a) Odd number of meat shields.

(b) Even number of unworthy sacrifices.

Figure 1: Examples for initial formation and placement on the circle for odd and even number of expendables, respectively.

Given a total number of horse fodder, n, the Coordinates for positions q_i and p_i are expressed as

$$x_{q_i} = \frac{1}{2}h \tag{1}$$

$$y_{q_i} = -\left[\left(\left\lceil\frac{n}{2}\right\rceil - (i+1)\right)h + d\right] \tag{2}$$

and

$$x_{p_i} = r \cdot \cos\varphi_i \tag{3}$$

$$y_{p_i} = r \cdot \sin\varphi_i + \gamma \tag{4}$$

.

For the odd case $\varphi_i = \vartheta_i + \frac{3}{2}\pi$, and for the even case $\varphi_i = \vartheta_i + \frac{3}{2}\pi + \frac{\pi}{n}$, with $\vartheta_i = \frac{2\pi}{n}i$. Setting the distance between two points p_i and p_{i+1} equal to λ, we find an expression for r:

$$r = \frac{\lambda}{\sqrt{\left(1 - \cos\left(\frac{2\pi}{n}\right)\right)^2 + \sin^2\left(\frac{2\pi}{n}\right)}} \tag{5}$$

Using the Théorème de Pythagore we get that the distance future-lance-kebab i has to move, l_i is given by

$$l_i = \sqrt{(x_{q_i} - x_{p_i})^2 + (y_{q_i} - y_{p_i})^2}. \tag{6}$$

Given $L = [|\gamma|, l_0, l_1, ..., l_n]$, a vector containing all the distances the units have to travel, we want the maximum value of L to be as small as possible, i.e. the optimal γ is given by:

$$\gamma_{opt} = \min_{\gamma}(\max L) \tag{7}$$

2.1 Magical results

Solving equation [7] would have been impossible for a starving peasant such as myself. Luckily a Damsel traveling through our blighted village generously bestowed upon us the gift of numerical results, nourishing our minds as well as our humility.

Using $h = 1m$, $\lambda = 1m$, $d = 2m$, she gave us the the example values displayed in table 1.

n	γ_{opt}
2	-1.00
3	-1.26
4	-1.25
5	-1.61
6	-1.57
7	-1.96
8	-1.90
9	-2.30
10	-2.23
11	-2.64
12	-2.57
13	-2.98
14	-2.90
15	-3.32
16	-3.24
17	-3.66
18	-3.58
19	-4.00
20	-3.92
21	-4.34
22	-4.26
23	-4.68
24	-4.60
25	-5.02
26	-4.94
27	-5.36
28	-5.28
29	-5.70
30	-5.62

Table 1: Example values for optimal γ, given n gruel-guzzlers.

3 discussion

Reading the values from table 1, we can see how far the chevalier has to move in order for the circle to be constructed in the fastest way possible, given our example values stated above. For example, given $n = 7$, we have that our Excellency must move a distance $\gamma = -1.96$, i.e. to the position of the first line of disappointments. For $n = 19$ we get $\gamma = -4.0$, i.e. his Lordship/her Ladyship must move to the third line of miscreants.

Now that this is all said and done, you can go ahead and forget all that you have read, as the optimal position of a pike circle is always where your liege has decided it should be.